Further Investigation on Chiral Symmetry Breaking in a Uniform External Magnetic Field

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Abstract

We study chiral symmetry breaking in QED when a uniform external magnetic field is present. We calculate higher order corrections to the dynamically generated fermion mass and find them to be small. In so doing we correct an error in the literature regarding the matrix structure of the fermion self-energy.

In Ref. [1] the effects of an external magnetic field on chiral symmetry breaking was studied using quantum electrodynamics as the model gauge field theory. The Schwinger-Dyson equation for the fermion self-energy in the quenched, ladder approximation was expressed in terms of Ritus' representation [2] for the exact fermion propagator in a constant magnetic field and an approximate solution for the dynamically generated fermion mass was found. The infrared fermion mass found in Ref. [1] was consistent with that obtained from other approaches [3].

In the present work we will examine how robust this approximate result is by carrying out the calculation to the next leading order of the approximation. In the course of our study, we discover an inconsistency in Ref. [1] regarding the form of the dynamical fermion mass matrix. We obtain the correct matrix structure and show that, despite this inconsistency, the infrared dynamical fermion mass found in Ref. [1] remains correct.

Let us begin with the Schwinger-Dyson equation for the fermion self-energy $\tilde{\Sigma}_A$ in the quenched, ladder approximation (see Eq. (34) in Ref. [1]):

$$\begin{aligned}\tilde{\Sigma}_A(\bar{p})\delta_{kk'} &= ie^2(2|eH|)\sum_{k''=0}^{\infty}\sum_{\{\sigma\}}\int\frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{i\mathrm{sgn}(eH)(n-n''+\tilde{n}''-n')\varphi}}{\sqrt{n!n'!n''!\tilde{n}''!}}\\ &\quad\cdot\,\mathrm{e}^{-\hat{q}_\perp^2}J_{nn''}(\hat{q}_\perp)J_{\tilde{n}''n'}(\hat{q}_\perp)\frac{1}{\hat{q}^2}\left(g_{\mu\nu}-(1-\xi)\frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}\right)\\ &\quad\cdot\,\Delta(\sigma)\gamma^\mu\Delta(\sigma'')\frac{1}{\gamma\cdot\bar{p}''+\tilde{\Sigma}_A(\bar{p}'')}\Delta(\tilde{\sigma}'')\gamma^\nu\Delta(\sigma').\end{aligned}\tag{1}$$

We adopt the notation and convention of Ref. [1] and will not explain them here. Since we are interested in finding a solution for the infrared fermion mass, the calculation will be simplified by considering the $\bar{p}\to 0$ limit in Eq. (1). Let us remind the reader that the 4-momentum $\bar{p}$ has the components $(p_0, 0, -\mathrm{sgn}(eH)\sqrt{2|eH|k}, p_3)$. Consider first the $k\to 0$ limit. Due to the presence of $\delta_{kk'}$ on the left hand side of Eq. (1), letting $k=0$ implies $k'=0$ which in turn requires $n=n'=0$ and $\sigma=\sigma'=\mathrm{sgn}(eH)$. This is because the quantum numbers n, k, and σ are related by the relation

$$n = n(k,\sigma) = k+\frac{\sigma}{2}\mathrm{sgn}(eH)-\frac{1}{2}\tag{2}$$

with the allowed values $n=0,1,2,...$, $k=0,1,2,...$, and $\sigma=\pm 1$, and similarly for $n'=n(k',\sigma')$. In this limit, the J polynomials in Eq. (1) become

$$\begin{aligned}J_{0n''}(\hat{q}_\perp) &= [i\ \mathrm{sgn}(eH)\hat{q}_\perp]^{n''},\\ J_{\tilde{n}''0}(\hat{q}_\perp) &= [i\ \mathrm{sgn}(eH)\hat{q}_\perp]^{\tilde{n}''}.\end{aligned}\tag{3}$$

and Eq. (1) now reads

$$\tilde{\Sigma}_A(\bar{p}_\parallel) \;=\; ie^2(2|eH|)\sum_{k''=0}^{\infty}\sum_{\sigma'',\tilde{\sigma}''}\int\frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{i\mathrm{sgn}(eH)(\tilde{n}''-n'')\varphi}}{\sqrt{n''!\tilde{n}''!}}\mathrm{e}^{-\hat{q}_\perp^2}[i\ \mathrm{sgn}(eH)\hat{q}_\perp]^{n''+\tilde{n}''}$$
$$\cdot\,\frac{1}{\hat{q}^2}\left(g_{\mu\nu}-(1-\xi)\frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}\right)\cdot\ \Delta\gamma^\mu\Delta(\sigma'')\frac{1}{\gamma\cdot\overline{p}''+\tilde{\Sigma}_A(\overline{p}'')}\Delta(\tilde{\sigma}'')\gamma^\nu\Delta \qquad (4)$$

where Δ is now understood to be $\Delta=\Delta(\mathrm{sgn}(eH))$, $\bar{p}_\parallel=(p_0,p_3)$, and the 4-momentum $\overline{p}''$ in the integrand is given by $\overline{p}''=\ (p_0-q_0,\,0,\,-\mathrm{sgn}(eH)\sqrt{2|eH|k''},\,p_3-q_3)$.

By transforming to polar coordinates for the integrals over $\hat{q}_1$ and $\hat{q}_2$: $\int d\hat{q}_1 d\hat{q}_2 = \int\hat{q}_\perp d\hat{q}_\perp d\varphi$, the integration over the angle φ can be carried out to yield

$$\int d\varphi\mathrm{e}^{i\mathrm{sgn}(eH)(\tilde{n}''-n'')\phi}=2\pi\delta_{\tilde{n}''n''}=2\pi\delta_{\tilde{\sigma}''\sigma''}. \qquad (5)$$

This allows us to rewrite Eq. (4) as

$$\tilde{\Sigma}_A(\bar{p}_\parallel) \;=\; ie^2(2|eH|)\sum_{k''=0}^{\infty}\sum_{\sigma''}\frac{1}{n''!}\int\frac{d^4\hat{q}}{(2\pi)^4}\mathrm{e}^{-\hat{q}_\perp^2}\frac{(-\hat{q}_\perp^2)^{n''}}{\hat{q}^2}$$
$$\cdot\,\left(g_{\mu\nu}-(1-\xi)\frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}\right)\Delta\gamma^\mu\Delta''\frac{1}{\gamma\cdot\overline{p}''+\tilde{\Sigma}_A(\overline{p}'')}\Delta''\gamma^\nu\Delta, \qquad (6)$$

where $\Delta''\ =\ \Delta(\sigma'')$.

The fermion self-energy is expected to have the form $\tilde{\Sigma}_A(\bar{p})=Z(\bar{p})\gamma\cdot\bar{p}+\Sigma_A(\bar{p})$, where $\Sigma_A(\bar{p})$ is a matrix representing the dynamically generated fermion mass. An ansatz was made in Ref. [1] that Σ_A was proportional to the unit matrix. An approximate solution for Σ_A was then obtained by keeping only the dominant $k''=0$ term on the right hand side of Eq. (6). In this case, $n''=0$ and $\sigma''=\mathrm{sgn}(eH)$. However, in the calculation of Ref. [1], the spin summations in Eq. (1) were carried out before the infrared limit was taken (see Eqs.(44)-(47) there), and contributions from both $\sigma''=\mathrm{sgn}(eH)$ and $\sigma''=-\mathrm{sgn}(eH)$ were included for the $k''=0$ term, thus leading to the incorrect conclusion that Σ_A was proportional to the unit matrix. We shall see below that Σ_A should be proportional to the matrix $\Delta(\mathrm{sgn}(eH))$, which is equal to $diag(1,0,1,0)$ for $\mathrm{sgn}(eH)=+1$ and equal to $diag(0,1,0,1)$ for $\mathrm{sgn}(eH)=-1$.

If we let $\Sigma_A(\bar{p})=m(\bar{p})\Delta$, where $m(\bar{p})$ denotes the dynamically generated fermion mass, and keep only the $k''=0$ contributions, Eq. (6) becomes

$$Z(\bar{p}_\parallel)\gamma\cdot\bar{p}_\parallel+m(\bar{p}_\parallel)\Delta \;\simeq\; ie^2(2|eH|)\int\frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\left(g_{\mu\nu}-(1-\xi)\frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}\right)$$
$$\cdot\,\Delta\gamma^\mu\Delta\frac{1}{[1+Z(\overline{p}''_\parallel)]\gamma\cdot\overline{p}''_\parallel+m(\overline{p}''_\parallel)\Delta}\Delta\gamma^\nu\Delta, \qquad (7)$$

where $\overline{p}''_\parallel = \bar{p}_\parallel - q_\parallel$. Because $\gamma\cdot\overline{p}''_\parallel$ commutes with Δ, $(-[1+Z]\gamma\cdot\overline{p}''_\parallel+m\Delta)([1+Z]\gamma\cdot\overline{p}''_\parallel+m\Delta) = [1+Z]^2(\overline{p}''_\parallel)^2+m^2\Delta$, which is a diagonal matrix that also commutes with $\gamma^\mu_\parallel = (\gamma^0,\gamma^3)$. We can therefore rewrite Eq. (7) as

$$\begin{aligned} Z(\bar{p}_\parallel)\gamma\cdot\bar{p}_\parallel + m(\bar{p}_\parallel)\Delta \quad &\simeq \quad ie^2(2|eH|)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\left(g_{\mu\nu} - (1-\xi)\frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}\right) \\ &\quad \cdot\ \Delta\gamma^\mu\Delta\frac{-[1+Z(\overline{p}''_\parallel)]\gamma\cdot\overline{p}''_\parallel + m(\overline{p}''_\parallel)\Delta}{[1+Z(\overline{p}''_\parallel)]^2(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)\Delta}\Delta\gamma^\nu\Delta. \end{aligned} \tag{8}$$

If we work in the Feynman gauge ($\xi = 1$) and use the property that

$$\Delta\gamma^\mu\Delta = \Delta\gamma^\mu_\parallel\Delta = \gamma^\mu_\parallel\Delta = \Delta\gamma^\mu_\parallel, \tag{9}$$

we find

$$\begin{aligned} &\Delta\gamma^\mu\Delta\frac{-[1+Z(\overline{p}''_\parallel)]\gamma\cdot\overline{p}''_\parallel + m(\overline{p}''_\parallel)\Delta}{[1+Z(\overline{p}''_\parallel)]^2(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)\Delta}\Delta\gamma_\mu\Delta \\ = \ &\frac{-2m(\overline{p}''_\parallel)\Delta}{[1+Z(\overline{p}''_\parallel)]^2(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)\Delta} \\ = \ &\frac{-2m(\overline{p}''_\parallel)}{[1+Z(\overline{p}''_\parallel)]^2(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)}\Delta \end{aligned} \tag{10}$$

and Eq. (8) becomes

$$Z(\bar{p}_\parallel)\gamma\cdot\bar{p}_\parallel + m(\bar{p}_\parallel)\Delta \ \simeq \ -ie^2(2|eH|)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\frac{2m(\overline{p}''_\parallel)}{[1+Z(\overline{p}''_\parallel)]^2(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)}\Delta. \tag{11}$$

It follows that $Z(\bar{p}_\parallel)$ vanishes in the Feynman gauge, as was found in Ref. [1], and one obtains the Schwinger-Dyson equation for the dynamical fermion mass:

$$m(\bar{p}_\parallel)\Delta \ \simeq \ -ie^2(2|eH|)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\frac{2m(\overline{p}''_\parallel)}{(\overline{p}''_\parallel)^2 + m^2(\overline{p}''_\parallel)}\Delta. \tag{12}$$

Note that both sides of this equation are proportional to the Δ matrix, which justifies our ansatz for the matrix structure of Σ_A.

Following the authors of Ref. [1], we seek a solution for m in the infrared limit ($\bar{p}_\parallel \to 0$) and approximate $m(\overline{p}''_\parallel)$ that appears in the integrand on the right hand side of Eq. (12) by its infrared value. This leads to the gap equation

$$1 \simeq e^2(4|eH|)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\frac{1}{2|eH|\hat{q}_\parallel^2 + m^2} \tag{13}$$

where we have made a Wick rotation to Euclidean space. This is precisely the gap equation (52) in Ref. [1], which has a solution of the form

$$m_0 \simeq a\sqrt{|eH|}\mathrm{e}^{-b\sqrt{\frac{\pi}{\alpha}}} \tag{14}$$

where a and b are real positive constants of order one and α is the fine structure constant. The subscript 0 indicates that this is the lowest order approximation for m. As noted in Ref. [1], this solution is applicable for small α.

Our result confirms the finding for the lowest order solution to the Schwinger-Dyson equation obtained in Ref. [1]. We also obtain the correct matrix structure for Σ_A, which is proportional to the matrix $\Delta(\mathrm{sgn}(eH))$.

Let us consider now the higher order corrections for the dynamical fermion mass m. In the Feynman gauge, $Z(\bar{p}_\|) = 0$ and the Schwinger-Dyson equation (6) may be written in the infrared limit as

$$\begin{aligned} m\Delta \;=\; & m_0\Delta + ie^2(2|eH|)\sum_{k''=1}^{\infty}\sum_{\sigma''}\frac{1}{n''!}\int\frac{d^4\hat{q}}{(2\pi)^4}\mathrm{e}^{-\hat{q}_\perp^2}\frac{(-\hat{q}_\perp^2)^{n''}}{\hat{q}^2} \\ & \cdot\,\Delta\gamma^\mu\Delta''\frac{1}{\gamma\cdot\overline{p}''+m(\overline{p}'')\Delta}\Delta''\gamma_\mu\Delta, \end{aligned} \tag{15}$$

where we have set $\Sigma_A(0) = m\Delta$ and separated the $k'' = 0$ term which contributes to the lowest order solution m_0. We shall estimate the next order corrections to m by replacing $m(\overline{p}'')$ in the integrand with the lowest order solution m_0. We need therefore to evaluate

$$\begin{aligned} (m-m_0)\Delta \;\simeq\; & ie^2(2|eH|)\sum_{k''=1}^{\infty}\sum_{\sigma''}\frac{1}{n''!}\int\frac{d^4\hat{q}}{(2\pi)^4}\mathrm{e}^{-\hat{q}_\perp^2}\frac{(-\hat{q}_\perp^2)^{n''}}{\hat{q}^2} \\ & \cdot\,\Delta\gamma^\mu\Delta''\frac{1}{\gamma\cdot\overline{p}''+m_0\Delta}\Delta''\gamma_\mu\Delta, \end{aligned} \tag{16}$$

where $\overline{p}'' = (-q_0,\, 0,\, -\mathrm{sgn}(eH)\sqrt{2|eH|k''},\, -q_3)$.

We shall consider the corrections to m coming from the following contributions. First, for $n'' = 0$, there is a subdominant $k'' = 1$ (and hence $\sigma'' = -\mathrm{sgn}(eH)$) term which was not included in m_0. Second, there are two terms for $n'' = 1$: one for $k'' = 1$ and $\sigma'' = \mathrm{sgn}(eH)$, the other for $k'' = 2$ and $\sigma'' = -\mathrm{sgn}(eH)$.

For $k'' \neq 0$, $(\gamma\cdot\overline{p}'')$ no longer commutes with Δ, and the expression for $(\gamma\cdot\overline{p}''+m_0\Delta)^{-1}$ is no longer so simple and depends on $\mathrm{sgn}(eH)$. We shall present our calculation below for the case $\mathrm{sgn}(eH) = +1$. However, the final result is applicable for either sign. For

$\mathrm{sgn}(eH) = +1$, one finds that $[\gamma \cdot \overline{p}'' + m_0\Delta(1)]^{-1}$ is given by

$$\frac{1}{(\lambda\beta-\kappa^2)^2-\lambda\beta m_0^2}\begin{pmatrix} -\lambda\beta m_0 & i\kappa\beta m_0 & \beta(\lambda\beta-\kappa^2) & i\kappa(\lambda\beta-\kappa^2) \\ i\kappa\lambda m_0 & \kappa^2 m_0 & -i\kappa(\lambda\beta-\kappa^2) & \lambda(\lambda\beta-\kappa^2-m_0^2) \\ \lambda(\lambda\beta-\kappa^2) & -i\kappa(\lambda\beta-\kappa^2) & -\lambda\beta m_0 & -i\kappa\lambda m_0 \\ i\kappa(\lambda\beta-\kappa^2) & \beta(\lambda\beta-\kappa^2-m_0^2) & -i\kappa\beta m_0 & \kappa^2 m_o \end{pmatrix}$$

where $\kappa \equiv \sqrt{2|eH|k''}$, $\lambda \equiv q_0 + q_3$, and $\beta \equiv q_0 - q_3$. Fortunately, this simplifies a great deal after we compute $\Delta\gamma^\mu\Delta''(\gamma \cdot \overline{p}'' + m_0\Delta)^{-1}\Delta''\gamma_\mu\Delta$.

Since $\sigma'' = \pm 1$, and

$$\Delta(1)\gamma_\perp^j\Delta(1) = 0 = \Delta(1)\gamma_\|^\mu\Delta(-1), \tag{17}$$

where $\gamma_\perp^j = (\gamma^1, \gamma^2)$, we need only evaluate

$$\begin{aligned} &\Delta(1)\gamma_\|^\mu\Delta(1)[\gamma\cdot\overline{p}''+m_0\Delta(1)]^{-1}\Delta(1)\gamma_{\|\mu}\Delta(1) \\ =\ &\gamma_\|^\mu\Delta(1)[\gamma\cdot\overline{p}''+m_0\Delta(1)]^{-1}\Delta(1)\gamma_{\|\mu} \\ =\ &\frac{2\lambda\beta m_0}{(\lambda\beta-\kappa^2)^2-\lambda\beta m_0^2}\cdot\Delta(1) \end{aligned} \tag{18}$$

and

$$\begin{aligned} &\Delta(1)\gamma_\perp^j\Delta(-1)[\gamma\cdot\overline{p}''+m_0\Delta(1)]^{-1}\Delta(-1)\gamma_{\perp j}\Delta(1) \\ =\ &\gamma_\perp^j\Delta(-1)[\gamma\cdot\overline{p}''+m_0\Delta(1)]^{-1}\Delta(-1)\gamma_{\perp j} \\ =\ &\frac{-2\kappa^2 m_0}{(\lambda\beta-\kappa^2)^2-\lambda\beta m_0^2}\cdot\Delta(1) \\ &+\frac{2(\lambda\beta-\kappa^2-m_0^2)}{(\lambda\beta-\kappa^2)^2-\lambda\beta m_0^2}\begin{pmatrix} 0 & \frac{\beta}{2}(1+\sigma_3) \\ \frac{\lambda}{2}(1+\sigma_3) & 0 \end{pmatrix} \end{aligned} \tag{19}$$

where σ_3 denotes the Pauli matrix. The off-diagonal elements in Eq. (19) might seem to be a problem. However, more careful inspection shows that they are odd in q_0 and q_3 and will vanish upon integration over these variables, thus reducing the matrix to a diagonal matrix proportional to $\Delta(1)$. It should be stressed that both matrix structures appearing in Eqs.(18) and (19) are proportional to $\Delta(1)$, consistent with the left-hand-side of Eq. (16). This shows that Σ_A is proportional to the matrix $\Delta(\mathrm{sgn}(eH))$ for all higher order terms in Eq. (15).

Putting all the pieces together, using Eq. (18) for the $k'' = 1$, $\sigma'' = 1$ term and Eq. (19) for the other two terms corresponding to $\sigma'' = -1$ and $k'' = 1, 2$, we find the next order

corrections for the dynamical fermion mass to be

$$
\begin{aligned}
(m-m_0)\Delta \;\simeq\; & ie^2(2|eH|)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{\mathrm{e}^{-\hat{q}_\perp^2}}{\hat{q}^2}\left\{\frac{-4|eH|m_0}{(-q_0^2+q_3^2+2|eH|)^2+m_0^2(-q_0^2+q_3^2)}\right. \\
& +\hat{q}_\perp^2\left[\frac{2m_0(-q_0^2+q_3^2)}{(-q_0^2+q_3^2+2|eH|)^2+m_0^2(-q_0^2+q_3^2)}\right. \\
& \left.\left.+\frac{8|eH|m_0}{(-q_0^2+q_3^2+4|eH|)^2+m_0^2(-q_0^2+q_3^2)}\right]\right\}\Delta. \qquad (20)
\end{aligned}
$$

In other words,

$$
\begin{aligned}
\frac{m-m_0}{m_0} \;\simeq\; & \frac{\alpha}{\pi}|eH|\int_0^\infty d\hat{q}_\perp^2\int_0^\infty d\hat{q}_\parallel^2\frac{e^{-\hat{q}_\perp^2}}{\hat{q}_\parallel^2+\hat{q}_\perp^2}\left\{\frac{1}{2|eH|(1+\hat{q}_\parallel^2)^2+m_0^2\hat{q}_\parallel^2}\right. \\
& \left.-\hat{q}_\perp^2\left[\frac{\hat{q}_\parallel^2}{2|eH|(1+\hat{q}_\parallel^2)^2+m_0^2\hat{q}_\parallel^2}+\frac{2}{2|eH|(2+\hat{q}_\parallel^2)^2+m_0^2\hat{q}_\parallel^2}\right]\right\} \qquad (21)
\end{aligned}
$$

where we have performed a Wick rotation to Euclidean space and carried out the integration over the polar angles on the $\hat{q}_\parallel$ plane as well as on the $\hat{q}_\perp$ plane.

We see that the fractional correction to m is quite small, being proportional to (α/π). If we substitute the expression for m_0 found in Eq. (14), with a and b set to be one, we can evaluate the integrals numerically to obtain the estimate

$$
\begin{aligned}
\frac{m-m_0}{m_0} \;&\simeq\; \left(\frac{\alpha}{\pi}\right)(0.4275-0.2161-0.1628) \\
&\simeq\; 0.0486\left(\frac{\alpha}{\pi}\right), \qquad (22)
\end{aligned}
$$

a very small number indeed.

In summary, we have reexamined the Schwinger-Dyson equation analysis of chiral symmetry breaking in QED in the presence of a uniform external magnetic field, confirming the lowest order result for the dynamical fermion mass obtained in Ref. [1], and finding the correct matrix structure for the fermion self-energy. We have also calculated the higher order corrections to the dynamical fermion mass and found them to be small, thus validating the approximation scheme used in the calculation.

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[1] D.-S. Lee, C. N. Leung, and Y. J. Ng, Phys. Rev. D **55**, 6504 (1997); C. N. Leung, Y. J. Ng, and A. W. Ackley, Phys. Rev. D **54**, 4181 (1996).

[2] Contribution of V. I. Ritus in *Issues in Intense-Field Quantum Electrodyanamics*, ed. V. L. Ginzburg (Nova Science, Commack, 1987).

[3] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D **52**, 4747 (1995); Nucl. Phys. **B462**, 249 (1996).